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On the stability of stablecoins

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ABSTRACT

This paper investigates the volatility processes of stablecoins and their potential stochastic interdependencies with Bitcoin volatility. We employ a novel approach to choose the optimal combination for the power law exponent and the minimum value for the volatilities bending the power law. Our results indicate that Bitcoin volatility is well-behaved in a statistical sense with a finite theoretical variance. Surprisingly, the volatilities of stablecoins are statistically unstable and contemporaneously respond to Bitcoin volatility. Also, whereas the volatilities of stablecoins are not Granger-causal for Bitcoin volatility, lagged Bitcoin volatility exhibits Granger-causal effects on the volatilities of stablecoins. We conclude that Bitcoin volatility is a fundamental factor that drives the volatilities of stablecoins.

1. Introduction

Bitcoin has a number of unique advantages over traditional payment methods, such as user autonomy, discretion, peer-to-peer focus, elimination of banking fees, low transaction fees for international payments, mobile payments, and 24/7 accessibility. However, these advantages come at the cost of volatility that well exceeds the volatility of many other asset classes. Baur et al. (2018) compared the statistical features of Bitcoin and other assets and found that the level of Bitcoin's historical return and volatility are not comparable to any other asset. The authors observed that Bitcoin has larger negative skewness than high yield corporate bond, gold, and silver returns. This large negative skewness was due to an asymmetric Bitcoin return distribution with fatter left-side than right-side tails. Very high kurtosis implies a large number of tail events in Bitcoin returns. Relevant to the present study, no previous studies investigate Bitcoin volatility and tail events.

In a speech at the 2018 World Economic Forum in Davos, Deputy Governor of the Swedish Central Bank Cecilia Skingsley commented that, unlike money, cryptocurrencies do not store value, fluctuate in value, and have unstable exchange rates.[3] Due

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[3] See https://www.weforum.org/agenda/2018/01/robert-shiller-bitcoin-is-just-an-interesting-experiment/.

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to Bitcoin's failure to serve as a substitute for national (fiat)[4] currencies, stablecoins have been developed as a possible solution. Stablecoins are designed to minimize price volatility by means of: (*i*) pegging against a national currency or commodity, (*ii*) collateralization with respect to other cryptocurrencies, or (*iii*) algorithmic coin supply management. In this regard, the most common type of stablecoin is the U.S. dollar-pegged coin Tether (USDT). On January 4, 2021 the Office of the Comptroller of the Currency (OCC) Chief Counsel issued a letter that legally approved payment-related activities involving new technologies for national banks and federal savings associations, including the use of independent node verification networks (INVNs or networks) and stablecoins.[5] The natural question that arises is whether this blockchain-based technology can provide stable currency payments. Seminal work by Wei (2018) examined whether the minting of new Tether stablecoins inflated the prices of Bitcoin. The author showed that, contrary to investor expectations, Tether issuances did not impact Bitcoin returns. However, a recent study by Griffin and Shams (2020) employed algorithms to explore blockchain data and found that purchases of Tether were timed following market downturns and resulted in large increases in Bitcoin prices. In another recent paper, Baur and Hoang (2021a) proposed a framework to test for absolute and relative stability of stablecoins. The authors argued that stablecoins are more stable than Bitcoin but less stable than stable benchmarks such as major national currencies.

The present paper proposes a new approach to measuring the stability of stablecoins. Given that Baur et al. (2018) found that Bitcoin exhibits extremely high kurtosis with relatively more tail events compared to other assets, we employ realized daily volatility to explicitly model the probability density functions of five stablecoins that exhibit the largest market capitalizations. For comparison purposes, we also model Bitcoin volatility using the same methodology. We apply power laws and maximum likelihood estimation (MLE) to estimate the corresponding power law exponents of the realized volatility processes.[6] A novel aspect of our approach is that it does not rely on the minimum value of the Kolmogorov–Smirnov distance measure used in earlier research. Instead of choosing the minimum value of the realized volatility ($\hat{x}_{MIN}$) required in MLE via that approach, we use a goodness-of-fit test in trial-and-error analyses to find the combination of $\hat{x}_{MIN}$ and corresponding power law exponent ($\hat{\alpha}$) for which the power law null hypothesis cannot be rejected. We demonstrate that our approach yields combinations of $\hat{x}_{MIN}$ and $\hat{\alpha}$ that are in line with the theoretical data generating process.

After evaluating realized volatilities, we explore stochastic interdependencies in volatilities. To do this, we utilize a log–log regression approach for both single and multiple equation models to test whether: (*i*) Bitcoin volatility and the volatilities of stablecoins are contemporaneously co-moving, (*ii*) lagged stablecoin volatilities have an impact on Bitcoin volatility, (*iii*) and lagged Bitcoin volatility has an impact on stablecoin volatility.

Our study contributes to previous literature in a number of important ways. First, we extend previous studies on tail risks associated with man-made phenomena. Often-cited work by Clauset et al. (2009) analyzed whether 24 real-world data sets from a range of different disciplines follow power law distributions. The authors' findings supported Taleb's (2007) view that power law distributions occur in many situations and help to better understand man-made phenomena. Another well-known study by Gabaix (2009) documented that income and wealth, the size of cities and firms, stock market returns, trading volume, international trade, and executive pay appear to follow different power law processes. Our study contributes to this literature by exploring uncertainty in cryptocurrency markets, which is a man-made phenomenon. It is worthwhile noting that the overall market capitalization of the cryptocurrency market is \$1.93 trillion[7]; hence, this market is not trivial in terms of economic significance.

From the perspective of finance research, power law distributions are used to model both financial asset returns and volatilities. A recent paper from Warusawitharana (2018) estimated power law coefficients using 15-minute stock returns for 41 stocks in the period 2003 to 2014. The results confirmed earlier findings by Plerou et al. (1999) by demonstrating that the power law coefficient of the cross-sectional distribution ranges between 2.09 and 3.46. Also, Liu et al. (1999) showed that the asymptotic behavior of the probability density function of the S&P 500 index is described by a power law distribution with an exponent around 4.[8] Extending these studies, we apply power laws to model the realized volatilities of cryptocurrencies and examine stochastic interdependencies in their volatility processes.

As mentioned earlier, the valuation of stablecoins is closely related to the valuation of national currencies (or fiat money) under a fixed exchange rate regime. Models for national currency valuation can be divided into those with macro- versus micro-foundations.[9] Macro-factor exchange rate models are based on country differences in money supplies, interest rates, capital flows, financial frictions, commodity prices, and inflation.[10] However, these models may not be relevant to the valuation of stablecoins that are not generally accepted as mediums of exchange by central banks in monetary policy. With respect to micro-foundation models, Lyons and Viswanath-Natraj (2019) found that fundamental factors such as order flows are valuation determinants of stablecoins. They also found that parity deviations of Tether (for example) are strongly affected by Bitcoin volatility. Our paper extends their analyses by focusing on the interconnections of volatilities between Bitcoin and stablecoin markets.

Additionally, our study contributes to the growing literature on emerging digital ecosystems. Wei (2018) examined the largest stablecoin Tether and its influence on Bitcoin. While he found no price manipulation of Bitcoin, as already mentioned, Griffin

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4 Similar to Lyons and Viswanath-Natraj (2019), we use the term *national currencies* instead of *fiat currencies* (paper-money) when referring to the analysis of stablecoins. Due to their collateral requirements and other characteristics, we treat the pricing of stablecoins in the spirit of valuation models of national currencies under a fixed exchange rate regime.

5 See https://www.occ.gov/news-issuances/news-releases/2021/nr-occ-2021-2a.pdf.

6 Taleb (2020), p.91) has commented that power laws should be the norm in modeling stochastic processes.

7 See coinmarketcap.com (as of August 18, 2021.)

8 See Lux and Alfarano (2016) for an excellent overview of power law distribution applications in finance.

9 See Lyons (2001) and Evans (2011) for standard text-book presentations covering these models.

10 For example, see studies by Eichengreen et al. (1994), Chen and Rogoff (2003), Engel and West (2005), Gabaix and Maggiori (2015), Itskhoki and Mukhin (2017), among others.

and Shams (2020) demonstrated that purchases of Tether are timed following market downturns and result in notable increases in Bitcoin prices. Also, other studies of stablecoins have analyzed their safe haven properties (Baur and Hoang, 2021b) and their pricing mechanisms in view of fixed exchange rate regimes for national currencies (Lyons and Viswanath-Natraj, 2019) and associated stability (Baur and Hoang, 2021a). While empirically our paper is closest to Baur and Hoang (2021a), unlike their study, we focus on: (*i*) uncertainty in stablecoin markets using realized volatilities, and (*ii*) stochastic interdependencies between the volatility processes of stablecoins and Bitcoin from a Granger-causal perspective.

Finally, there is a large literature on modeling the volatilities of cryptocurrency data using various Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model specifications.[11] In a recent study, Caporale and Zekokh (2019) fitted more than 1000 GARCH-type models to the log-returns of Bitcoin, Ethereum, Ripple and Litecoin. Their results suggested that using standard GARCH models may yield incorrect Value-at-Risk (VaR) and Expected Shortfall (ES) forecasts because cryptocurrency data exhibits a high level of asymmetries and regime switches. The authors argued that using GARCH models results in ineffective risk-management, portfolio optimization, and pricing of derivative securities.[12]

In this regard, Taleb (2020, pp. 50–51) has stressed that the application of GARCH models is problematic if either the data do not exhibit finite kurtoses or if the kurtoses are not defined. Specifically, given the kurtosis is infinite, estimates of GARCH models may be sample specific. By contrast, Calvet and Fisher (2004) and Lux et al. (2014) showed that power law models usually outperform GARCH models in terms of forecasting financial market volatility.

It is interesting to note that Hou et al. (2020) documented that about 80 percent of 452 cross-sectional asset pricing anomalies fails scientific replications. In an earlier contribution, Schwert (2003) pointed out that, once cross-sectional asset pricing phenomena are documented and analyzed in the academic literature, these cross-sectional patterns often seem to disappear, reverse, or attenuate. He explained that asset pricing anomalies could be subject to statistical aberrations, which have attracted the attention of academics and practitioners. Hence, sample specificity could affect the results. Our study contributes to the literature on the volatility of cryptocurrencies by modeling the realized volatilities of cryptocurrencies using power laws. This approach allows (*i*) modeling of 'wild volatility' often observed in cryptocurrency markets, and (*ii*) determination of whether or not distribution-specific metrics are subject to sample-specificity.[13]

Our results demonstrate that the volatility processes of both Bitcoin and stablecoins bend power laws. Surprisingly, Bitcoin volatility is rather well-behaved in terms of being a finite and statistically stable process. The power law exponent for Bitcoin volatility conforms to a theoretical variance. However, we do not find such evidence for the volatility processes of stablecoins. While our results suggest that theoretical means of stablecoin volatility processes exist, their corresponding variances are infinite, which indicates statistical instability. Furthermore, using log–log regression analysis to explore potential volatility spillover effects shows that uncertainties in stablecoins and Bitcoin respond contemporaneously. However, the volatilities of stablecoins (including Tether) do not spillover to Bitcoin volatility. Lastly, we find strong volatility spillover effects in a Granger-causal sense from Bitcoin volatility to volatilities of stablecoins. These results support those of Lyons and Viswanath-Natraj (2019) on the role of Bitcoin volatility as a fundamental factor affecting the volatilities of stablecoins.

The paper is organized as follows. Section 2 provides background discussion. Section 3 presents the results of our statistical analyses. Section 4 discusses the empirical results. The last section concludes.

2. Background discussion

While Bitcoin is the most popular cryptocurrency, it suffers from high volatility imposing high risk on investors. Even intraday price fluctuations are huge. It is no surprise to see Bitcoin's price moving in excess of 10% in either direction on a daily basis.[14] Even long-term price fluctuations are very high — for instance, Bitcoin's price rose from the level of around $7,500 in November 2017 to about $20,000 in December 2017, and then declined to $4,200 in December 2018. Recently, over-the-counter (OTC) investor interest in Bitcoin caused the price to surge almost 200% reaching $22,800 in December 2020. Due to large volatility and periodic collapses in the Bitcoin market, investors started looking for a less volatile crypto-asset known as *stablecoins*.

Even before the advent of stablecoins, the rapid transformation of payments from a cash to digital ecosystem prompted interest in the issuance of central bank digital currencies. In 2020 the Federal Reserve announced an investigation into its own digital currency with potential stability comparable to the U.S. dollar. There are two main reasons for the price stability of national currencies. First, national currencies are backed by relatively stable underlying or collateral assets, such as gold or forex reserves. And second, when a national currency's price moves beyond a certain level, central bank authorities can act to maintain price stability. By contrast, cryptocurrencies typically lack both of these supporting features. Consequently, interest in cryptocurrencies resembling the stability of national currencies helped to motivate the development of stablecoins.

There are three different types of stablecoins. First, fiat-collateralized (off-chain) stablecoins backed by a national currency (e.g., the U.S. dollar) as collateral for issuing tokens. The token is a 1:1 ratio of cryptocurrency/national money. Other forms of

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[11] In a recent study, Grobys (2021) provided an intensive review on this literature.

[12] Caporale and Zekokh's (2019) finding is supported by Ardia et al. (2019), who tested the presence of regime changes in the GARCH volatility dynamics of Bitcoin log-returns using Markov-switching GARCH (MSGARCH) models. Their findings indicated that MSGARCH models clearly outperform single-regime GARCH for VaR and ES forecasting.

[13] Some relevant studies on the applications of power laws for financial market data include Lux and Alfarano (2016), Lux and Ausloos (2002), Calvet and Fisher (2004), Lux et al. (2014), and Liu et al. (1999). Notably, Calvet and Fisher (2004) and Lux et al. (2014) showed that power law models often outperform GARCH models in terms of forecasting financial market volatility.

[14] For example, in the sample from March 29th, 2013 to November 22nd, 2020, Bitcoin's price increased (decreased) more than 10% (-10%) on 61 (53) trading days.

Table 1
Market capitalization.

Rank	Cryptocurrency	Abbreviation	Market capitalization	Data (MM/DD/YYYY)
1	Bitcoin	BTC	$340,790,183,317	4/29/2013–11/22/2020
4	Thether	USDT	$18,495,477,972	2/25/2015–11/22/2020
13	USD Coin	USDC	$2,871,675,203	10/8/2018–11/22/2020
26	Dai	DAI	$1,019,324,729	11/22/2019–11/22/2020
37	Binance USD	BUSD	$653,999,894	9/20/2019–11/22/2020
56	TrueUSD	TUSD	$314,313,722	3/6/2018–11/22/2020

Daily data were downloaded for Bitcoin (BTC), Thether (USDT), USD Coin (USDC), Dai (DAI), Binance USD (BUSD), and TrueUSD (TUSD) from coinmarketcap.com. Market capitalization, available time period, and rank for each cryptocurrency are as of November 22, 2020.

collateral are commodities, including precious metals such as gold and silver. Most fiat-collateralized stablecoins use U.S. dollar reserves. Some popular examples are Circle (USDC), Gemini Dollar (GUSD), and Tether (USDT). However, dollar-based stablecoins can differ in their reserves. Whereas USDC is backed by the USD in the ratio 1:1, USDT is backed by a basket of various U.S. reserves and assets. Off-chain fiat-collateralized stablecoins are created when national currency is held by a centralized issuer and destroyed when the fiat asset is received. Thus, these stablecoins seek to make transactions safe, fast, and secure for daily transactions. Some other advantages of fiat-collateralized stablecoins are convenience, simplicity, and (prima facie) stability. Disadvantages include the use of a centralized blockchain, which is vulnerable to the moral hazards and potential bankruptcy of the central authority.

Second, crypto-collateralized (on-chain) stablecoins are tokens backed by other cryptocurrencies. Generally, these stablecoins are backed by a portfolio of different cryptocurrencies for risk management in terms of diversification. They are often over-collateralized to mitigate the risk of price fluctuations of underlying cryptocurrencies. This characteristic implies that a considerable part of the token supply is maintained as a reserve in order to distribute a lower number of stablecoins – a mechanism allowing the issuers to maintain price stability. The most common form of crypto-supported stablecoin requires users to store a fixed deposit or stake a certain amount of digital currencies into a smart contract, known as a *Collateralized Debt Position* (CDP) or *Vault*. This requirement results in a fixed ratio of stablecoins. Typically, a decentralized blockchain provides trust, transparency, and security to users. However, there are some disadvantages, particularly the need for excess collateral. For instance, $1,000 worth of Ether may be held as reserves for issuing an equivalent of $500 worth of crypto-collateralized stablecoins. Another example is MakerDAO's DAI stablecoin generated when the investor opens a CDP, deposits some amount of Ether (ETH) as collateral, and then withdraws DAI from their Vault. Investors must maintain a collateralization ratio of 1.5, which means that investors must collateralize 150% of their DAI holdings. In simple terms, to take a loan of 100 DAI, investors have to deposit $150 worth of ETH into the CDP/Vault. This requirement ensures that the system has enough collateral to account for the whole DAI supply in circulation and maintain solvency. It should be noted that backing by multiple cryptocurrencies makes it difficult to achieve price stability. Since the majority of cryptocurrencies are in the same asset class and follow similar trends over time, a basket of cryptocurrencies is undiversified with little or no reduction in price stability.[15]

Third, and last, non-collateralized (seigniorage) stablecoins use the Seigniorage Shares System, wherein algorithms seek to maintain price stability without being backed by any national currency, physical asset, or cryptocurrency. This system algorithmically increases or decreases the supply of cryptocurrency in a manner similar to central bank quantitative easing or tightening. The objective of this mechanism is to maintain price stability as close to $1 U.S. dollar as possible by selling tokens if the price falls below the peg or supplying tokens to the market if the value increases. To do this, the cryptocurrency base coin uses a consensus mechanism to determine whether it should increase or decrease the supply of tokens. Non-collateralized cryptocurrencies form the minority group of stablecoins. Some examples are SagaCoin (SAGA), Havven (HAV), and Carbon (CUSD). The main advantage of non-collateralized stablecoins is that there is no reliance on collateral. As such, this type of stablecoin is independent from a central authority. Avoiding collateral also decreases other risks, such as bankruptcy and moral hazard related to centralization. On the other hand, this approach is far more complex than collateralized stablecoins, which makes it difficult to understand for naïve users.

3. Data and methodology

We download daily data for Bitcoin (BTC) as well as stablecoins Tether (USDT), USD Coin (USDC), Dai (DAI), Binance USD (BUSD), and TrueUSD (TUSD) from coinmarketcap.com. These data include all available opening, high, low, and closing prices over time. Table 1 summarizes the market capitalization, available time period, and rank for each cryptocurrency as of November 22, 2020. The data series for BTC (DAI) covers the longest (shortest) sample period from 4/29/2013–11/22/2020 (11/22/2019–11/22/2020).

[15] Stosic et al. (2018) observed the presence of multiple collective behaviors among cryptocurrencies. Also, Bouri et al. (2019) showed that the cryptocurrency market is subject to herding behavior that appears to vary over time.

Table 2
Descriptive statistics.

Metric/Cryptocurrency	BTC	USDT	USDC	DAI	BUSD	TUSD
Mean	0.5256	0.1670	0.2096	0.2722	0.2256	0.2303
Median	0.3712	0.1158	0.1951	0.2445	0.1982	0.1990
Maximum	7.3973	3.1091	1.8356	2.2510	2.8147	5.7823
Minimum	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.5525	0.2317	0.1849	0.2541	0.2372	0.2777
Skewness	3.9396	3.9485	2.7371	2.5324	4.5522	10.0989
Kurtosis	28.8858	36.4061	18.8971	15.4920	41.7549	175.3898
Observations	2765	2093	777	367	430	993

Descriptive statistics for realized annualized daily volatilities for Bitcoin (BTC), Tether (USDT), USD Coin (USDC), Dai (DAI), Binance USD (BUSD), and TrueUSD (TUSD) from coinmarketcap.com. Following Grobys (2021), realized annualized daily volatilities are computed for each cryptocurrency as proposed in Rogers and Satchell (1991), that is,

$$\sigma_{i,t} = \sqrt{T}\sqrt{\left(ln\left(\frac{HIGH_{i,t}}{CLOSE_{i,t}}\right)\cdot ln\left(\frac{HIGH_{i,t}}{OPEN_{i,t}}\right)+ln\left(\frac{LOW_{i,t}}{CLOSE_{i,t}}\right)\cdot ln\left(\frac{LOW_{i,t}}{OPEN_{i,t}}\right)\right)},$$

where $HIGH_{i,t}$, $LOW_{i,t}$, $OPEN_{i,t}$, and $CLOSE_{i,t}$ denote the highest, lowest, opening, and closing price for cryptocurrency i on day t, $\sigma_{i,t}$ denotes cryptocurrency i's corresponding realized annualized volatility, and $T = 365$ due to 24/7 cryptocurrency trading.

3.1. Realized volatility

We compute realized volatilities for each cryptocurrency. Following Grobys (2021), realized annualized daily volatilities are compounded in line with Rogers and Satchell (1991):

$$\sigma_{i,t} = \sqrt{T}\sqrt{\left(ln\left(\frac{HIGH_{i,t}}{CLOSE_{i,t}}\right)\cdot ln\left(\frac{HIGH_{i,t}}{OPEN_{i,t}}\right)+ln\left(\frac{LOW_{i,t}}{CLOSE_{i,t}}\right)\cdot ln\left(\frac{LOW_{i,t}}{OPEN_{i,t}}\right)\right)}, \tag{1}$$

where $HIGH_{i,t}$, $LOW_{i,t}$, $OPEN_{i,t}$, and $CLOSE_{i,t}$ denote the highest, lowest, opening, and closing price for cryptocurrency i on day t, respectively, $\sigma_{i,t}$ denotes cryptocurrency i's corresponding realized annualized volatility, and $T = 365$ as cryptocurrencies are traded 24/7. Table 2 reports the descriptive statistics, and Fig. 1 plots the time series evolution of the calculated realized cryptocurrency volatilities. In Table 2 we see that BTC exhibits the highest average volatility equal to 53%, whereas the stablecoins' average volatilities range between 17% (USDT) and 27% (DAI). Hence, BTC's average volatility is considerably higher than the average volatilities of stablecoins. An important empirical fact that we can observe from Fig. 1 is that each realized volatility series contains outliers. Statistically, this phenomenon can be measured by the kurtosis value. Table 2 shows that each cryptocurrency's volatility exhibits very high kurtosis ranging from 18.90 (USDC) to 175.39 (TUSD). For comparison purposes, the thin-tailed normal distribution has a kurtosis of 3. We infer that all cryptocurrency volatilities have extremely heavy fat tails.

3.2. Statistical model

To investigate the stability of volatility processes, we model the realized volatilities using the following power laws:

$$P(X > x) = p(x) = Cx^{-\alpha}, \tag{2}$$

where $C = (\alpha - 1)x_{MIN}^{\alpha-1}$ with $\alpha \in \{\mathbb{R}_+ \,|\alpha > 1\}$, $x \in \{\mathbb{R}_+ \,|x_{MIN} \le x < \infty\}$, x_{MIN} is the minimum value of realized volatility that bends the power law, and α is the magnitude of tail exponent.[16] Regarding the latter term, Taleb (2020, p. 34) observed that the tail exponent of a power law function captures via extrapolation the low-probability deviation not seen in the data, which plays a disproportionately large share in determining the mean. It can be shown that the expectation of the volatilities defined as $E[X]$ is given by

$$E[X] = \int_{x_{MIN}}^{\infty} xp(x)\,dx = \frac{(\alpha-1)}{(\alpha-2)}x_{MIN}, \tag{3}$$

and that the second moment $E[X^2]$, or the variance of the volatility, is defined as:

$$E[X^2] = \int_{x_{MIN}}^{\infty} x^2 p(x)\,dx = \frac{(\alpha-1)}{(\alpha-3)}x_{MIN}^2. \tag{4}$$

Higher moments of order k are analogously defined as:

$$E[X^k] = \frac{(\alpha-1)}{(\alpha-1-k)}x_{MIN}^k. \tag{5}$$

[16] We follow notation in Clauset et al. (2009). To keep our notations clear, we drop the index i denoting the respective realized volatility of an individual cryptocurrency. Volatilities are calculated separately for each cryptocurrency $i = 1, \ldots, 6$. In choosing power laws to model financial data, we follow Liu et al. (1999) among others.

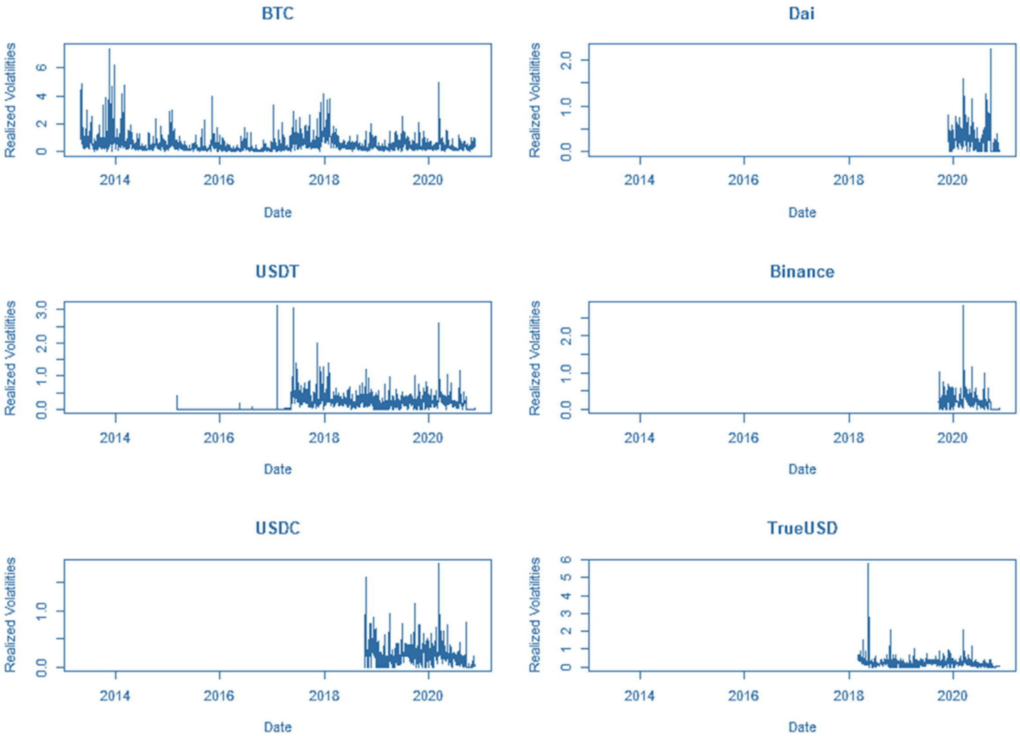

Fig. 1. Time series evolutions of realized volatilities.
This figure shows the time series evolutions for the realized annualized daily volatilities for Bitcoin (BTC), Tether (USDT), USD Coin (USDC), Dai (DAI), Binance USD (BUSD), and TrueUSD (TUSD). The realized annualized daily volatility for cryptocurrency i at time t is computed as,
$\sigma_{i,t} = \sqrt{T}\sqrt{\left(ln\left(\frac{HIGH_{i,t}}{CLOSE_{i,t}}\right)\cdot ln\left(\frac{HIGH_{i,t}}{OPEN_{i,t}}\right)+ln\left(\frac{LOW_{i,t}}{CLOSE_{i,t}}\right)\cdot ln\left(\frac{LOW_{i,t}}{OPEN_{i,t}}\right)\right)}$,
where $HIGH_{i,t}$, $LOW_{i,t}$, $OPEN_{i,t}$, and $CLOSE_{i,t}$ denote the highest, lowest, opening, and closing price for cryptocurrency i on day t, $\sigma_{i,t}$ denotes cryptocurrency i's corresponding realized annualized volatility, and $T = 365$ due to 24/7 cryptocurrency trading.

From Eq. (3), we know that the mean only exists for $\alpha > 2$, whereas the variance only exists for $\alpha > 3$. Following White et al. (2008) and Clauset et al. (2009), who found that maximum likelihood estimation (MLE) performs best for estimating power law exponents, we estimate the tail exponent as:

$$\hat{\alpha} = 1 + N\left(\sum_{i=1}^{N}\ln\left(\frac{x_i}{x_{MIN}}\right)\right)^{-1}, \tag{6}$$

where $\hat{\alpha}$ denotes the MLE estimator, N is the number of observations exceeding x_{MIN} and other notation is as before. Fig. 2 plots the estimated parameters for $\hat{\alpha}$ depending on the value for x_{MIN} for all of our cryptocurrency volatilities.[17] A crucial question is: How can we determine the corresponding values for α and x_{MIN} to accurately estimate the probability density functions? Clauset et al. pointed out that it is common to choose the value for x_{MIN}, where $\hat{x}_{MIN}$ denotes the selected value for x_{MIN}, beyond which $\hat{\alpha}$ is stable. From Figure 3 in Clauset et al. (2009, p. 670), it is evident that this value corresponds to the saddle point in a $\hat{\alpha}/\hat{x}_{MIN}$-graph. Clauset et al. proposed the Kolmogorov–Smirnov approach to choose the optimal value for $\hat{x}_{MIN}$. This statistic is simply the maximum distance D between the data and fitted CDFs defined as:

$$D = MAX_{x\geq x_{MIN}}\left|S(x) - P(x)\right|, \tag{7}$$

where $S(x)$ is the CDF of the data for the observation with value at least x_{MIN}, and $P(x)$ is the CDF for the power law model that best fits the data in the region $x \geq x_{MIN}$. The estimate of the x_{MIN} is the value of x_{MIN} that minimizes D. This approach may yield accurate estimates in the case of well-behaved $\hat{\alpha}/\hat{x}_{MIN}$-functions, such as illustrated in Figure 3 of Clauset et al. (2009, p. 670). However, it could lead to severe errors (as shown in forthcoming results) in the presence of erratic functions.[18] For example, we observe from Fig. 2 that the $\hat{\alpha}/\hat{x}_{MIN}$-function for BTC looks virtually the same as the $\hat{\alpha}/\hat{x}_{MIN}$-function for a simulated power law process in Figure 3 of Clauset et al. By contrast, the $\hat{\alpha}/\hat{x}_{MIN}$-functions for our stablecoins do not show this relatively smooth pattern but rather appear to be much more erratic.

[17] These graphs are often referred to as *Hill plots*.
[18] Due to finite samples in empirical research, this situation is not unexpected.

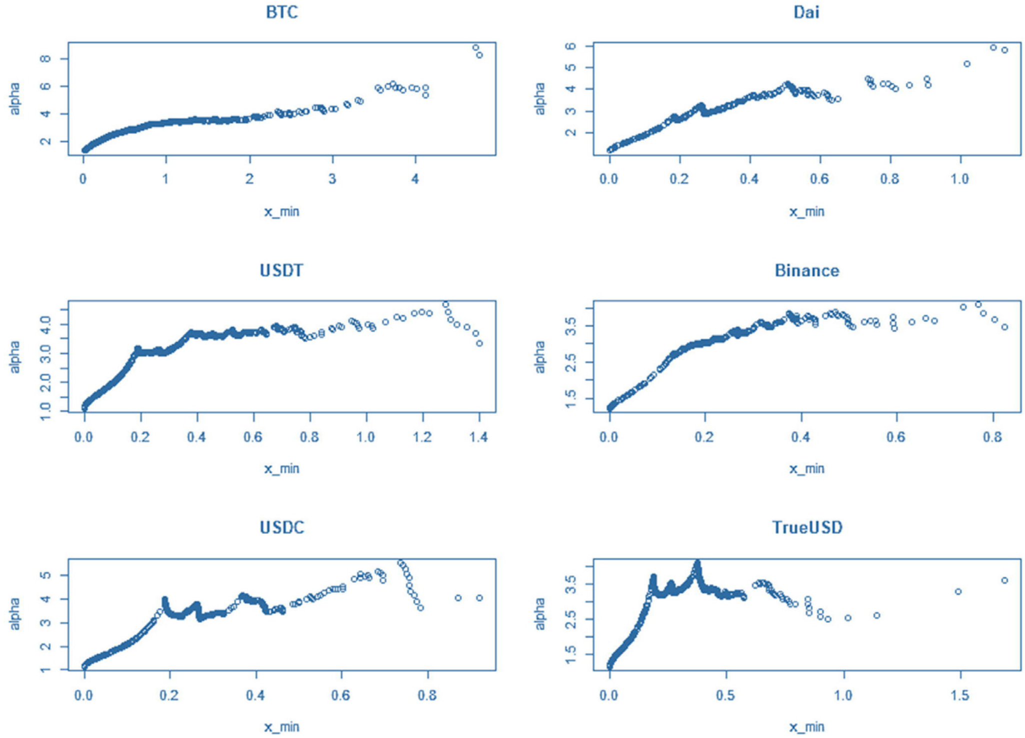

Fig. 2. Hill plots.
This figure shows the Hill plots for Bitcoin (BTC), Tether (USDT), USD Coin (USDC), Dai (DAI), Binance USD (BUSD), and TrueUSD (TUSD). For each cryptocurrency, the Hill plot shows the estimated $\hat{\alpha}$ as a function of x_{MIN} given by the maximum likelihood estimator (MLE),
$\hat{\alpha} = 1 + N\left(\sum_{i=1}^{N} \ln\left(\frac{x_i}{x_{MIN}}\right)\right)^{-1}$,
where $\hat{\alpha}$ denotes the MLE estimator, x_i is the realized annualized daily volatility of the respective cryptocurrency, provided $x_i \geq x_{MIN}$, and N denotes the number of observations for which $x_i \geq x_{MIN}$ is satisfied.

Therefore, instead of using the Kolmogorov–Smirnov approach as outlined above, we propose a different approach to choose the optimal combination of $\hat{\alpha}$ and $\hat{x}_{MIN}$ – namely, a combination of $(\hat{\alpha}, \hat{x}_{MIN})$, where the theoretical probability density function conforms to the empirical one. For each realized volatility series, we choose a parameter vector $\hat{\boldsymbol{\alpha}} = (2.00, 2.50, 3.00, 3.50)$ in association with the corresponding vector of $\hat{\mathbf{x}}_{MIN}$ (as observed in Fig. 2). Note that $\hat{\boldsymbol{\alpha}}$ is arbitrarily chosen and covers the space in which we expect to find a combination of $(\hat{\alpha}, \hat{x}_{MIN})$ wherein the power law null hypothesis is not rejected. Then, for each combination pair $(\hat{\alpha}, \hat{x}_{MIN})$, as defined by $(2.00, \hat{x}_{MIN,\ \hat{\alpha}=2.00})$, $(2.50, \hat{x}_{MIN,\ \hat{\alpha}=2.50})$, $(3.00, \hat{x}_{MIN,\ \hat{\alpha}=3.00})$, and $(3.50, \hat{x}_{MIN,\ \hat{\alpha}=3.50})$, we determine its specific distance D per equation (7) and then employ the goodness-of-fit test as detailed in Section 4.1. of Clauset et al. (2009, pp. 675–678). Under the null-hypothesis of this test, it is assumed that the data generating process follows a power law function with the corresponding combination $(\hat{\alpha}, \hat{x}_{MIN})$. Using a statistical significance level of 5%, we do not reject the power law null hypothesis if the p-value exceeds 5%.[19] Using $\hat{\boldsymbol{\alpha}} = (2.00, 2.50, 3.00, 3.50)$ in association with the corresponding vector of $\hat{\mathbf{x}}_{MIN}$ in our tests allows us to identify whether each volatility series exhibits a mean and/or variance, i.e., the mean (variance) only exists for $\alpha > 2$ ($\alpha > 3$).[20]

Considering USDT as an illustrative example, we observe from Table 3 that for the combinations $(\hat{\alpha} = 2.00, \hat{x}_{MIN} = 0.1011)$ and $(\hat{\alpha} = 2.50, \hat{x}_{MIN} = 0.1487)$ the power law null hypothesis cannot be rejected, whereas the null hypothesis is rejected for $(\hat{\alpha} = 3.00, \hat{x}_{MIN} = 0.1790)$ and $(\hat{\alpha} = 3.50, \hat{x}_{MIN} = 0.3554)$. This result implies that between $\hat{\alpha} = 3.00$ and $\hat{\alpha} = 2.50$ it is possible to find a combination $(\hat{\alpha}, \hat{x}_{MIN})$ for which the power law null hypothesis cannot be rejected. Here our proposed approach works as follows: On the space $\hat{\alpha} \in \{2.5001, 2.5002, \ldots, 2.9998, 2.9999\}$ with corresponding $\hat{x}_{MIN}$, we iteratively determine for each corresponding combination pair $(\hat{\alpha}, \hat{x}_{MIN})$ its specific distance D as defined in Eq. (7) and then employ Clauset et al.'s goodness-of-fit test. Specifically, moving from the combination $(\hat{\alpha} = 3.00, \hat{x}_{MIN} = 0.1790)$ to $(\hat{\alpha} = 2.50, \hat{x}_{MIN} = 0.1487)$, we make use of trial-and-error attempts to search for the combination $(\hat{\alpha}, \hat{x}_{MIN})$ for which we cannot reject the power law null hypothesis the first time. For

[19] As pointed out in Clauset et al. (2009), after estimating the KS statistic for each fit, a large number of power-law distributed synthetic data sets with $(\hat{\alpha}, \hat{x}_{MIN})$ equal to those of the distribution are generated. Each synthetic data set is individually fitted to its own power-law model, and the KS statistic is computed for each one relative to its own model. The fraction of time that the resulting statistic is larger than the value for the empirical data is counted, which is the corresponding p-value.

[20] See Eq. (3).

Table 3
Assessing the optimal power law exponent.

Exponent/Cryptocurrency	BTC	USDT	USDC	DAI	BUSD	TUSD
Panel A. Exponents and p-values.						
2.00	1.0000	1.0000	0.9978	1.0000	1.0000	1.0000
2.50	1.0000	1.0000	0.0004	0.5029	0.9934	1.0000
3.00	0.9977	0.0000	0.0000	0.0090	0.0004	0.0170
3.50	0.7494	0.0000	0.0000	0.0469	0.0044	0.0000
Panel B. Exponents and $\hat{x}_{\mathrm{MIN}}$ values.						
2.00	0.1824	0.1011	0.0895	0.1073	0.0954	0.0924
2.50	0.3591	0.1487	0.1318	0.1667	0.1414	0.1347
3.00	0.6947	0.1790	0.1579	0.2433	0.1699	0.1639
3.50	1.2846	0.3554	0.1864	0.3881	0.1882	0.1816
Panel C. Optimal estimates based on trial-and-error.						
$\hat{\alpha}$	3.4643	2.9024	2.3607	2.8699	2.8273	2.9123
$\hat{x}_{MIN}$	1.0950	0.1752	0.1224	0.2268	0.1595	0.1587
p-value	0.5027	0.3125	0.2325	0.0534	0.6000	0.6297

To find the optimal combination of $\hat{\alpha}$ and $\hat{x}_{MIN}$ (i.e., the combination that is most likely to have generated the underlying stochastic process of the data), for each volatility series a parameter vector $\hat{\boldsymbol{\alpha}} = (2.00, 2.50, 3.00, 3.50)$ is chosen in association with the corresponding vector of $\hat{\boldsymbol{x}}_{MIN}$ (see Fig. 2). Next, for each combination pair $\left(\hat{\alpha}, \hat{x}_{MIN}\right)$, the specific distance D as defined in Eq. (7) is determined and then the goodness-of-fit test is employed as discussed in Section 4.1. of Clauset et al. (2009, pp. 675–678). Under the null-hypothesis of this test, it is assumed that the data generating process follows a power law with the corresponding combination $\left(\hat{\alpha}, \hat{x}_{MIN}\right)$. Using a statistical significance level of 5%, the power law null hypothesis is not rejected if the p-value exceeds 5%.

instance, in the case of USDT, we cannot reject the power law null hypothesis for the combination $\left(\hat{\alpha} = 2.9024, \hat{x}_{MIN} = 0.1752\right)$, which results a p-value of 0.3125 for the goodness-of-fit test.[21]

In the same manner we evaluated the $\left(\hat{\alpha}, \hat{x}_{MIN}\right)$ for USDC, DAI, BUSD, and TUSD. Next, analyzing the volatility process of BTC, we also observe from Table 3 that for any combination of $\left(\hat{\alpha}, \hat{x}_{MIN}\right)$ the power law null hypothesis holds. Consequently, we can simply rely on the saddle point in the $\hat{\alpha}/\hat{x}_{MIN}$-graph for BTC (see Fig. 1), which reaches the optimum for the combination $\left(\hat{\alpha} = 3.4643, \hat{x}_{MIN} = 1.0950\right)$.

Our findings have a number of important implications. First, since $\hat{\alpha} > 3$, the volatility of BTC has both a theoretical mean and theoretical variance. In this regard, Taleb (2020, p. 50) noted: "If we don't know anything about the fourth moment, we know nothing about the stability of the second moment. It means we are not in a class of distribution that allows us to work with the variance, even if it exists". The same holds for the variance of the variance or the variance of the volatility.

If the second moment of a distribution does not exist, we know nothing about the stability of the first moment, such that we cannot make inference based on the mean even if it exists. Since Bitcoin volatility's theoretical variance exists, the mean of Bitcoin's volatility is stable. This finding has some important implications. For example, if the sample size is large enough, the mean of realized Bitcoin volatility converges towards its true value; hence, the mean is both computable and informative. This statistical stability is manifested in Bitcoin volatility's power law exponent > 3. However, this outcome is obviously not the case for stablecoins. From Table 3 we observe that the second moments for none of the realized stablecoins' volatilities exist. Infinite variances imply that the true mean of realized volatilities is unobservable in finite samples. This statistical instability is manifested in stablecoins volatilities' power law exponents < 3. From a practical point of view, this statistical instability is manifested in huge spikes in the volatility processes for stablecoins (see Fig. 1), which become increasingly larger across time, even though their occurrence may be less frequent.

Given the large literature on GARCH-type modeling of cryptocurrencies' volatilities, it is reasonable to investigate the difference between GARCH models, realized volatilities, and power laws. Using a standard GARCH(1,1) model, which is often used as a benchmark model in empirical finance research, we employ the log-returns of cryptocurrency i denoted here as $crypto_i$ and estimate the following model:

$$crypto_{i,t} = a_i + e_{i,t},$$

$$\sigma^2_{i,t} = b_{i,0} + b_{i,1} e^2_{i,t-1} + b_{i,2} \sigma^2_{i,t-1},$$

$$e_{i,t} = \sqrt{\sigma^2_{i,t}} \epsilon_{i,t},$$

where a_i denotes the intercept for the mean equation, $b_{i,0}$, $b_{i,1}$, $b_{i,2}$ denote the parameters for the variance equation, $e_{i,t}$ denotes the residual term at time t for the mean equation for $crypto_{i,t}$, and $\sigma^2_{i,t}$ denotes the conditional variance at time t. This model can be estimated via MLE in which it is typically assumed that the innovation process $\epsilon_{i,t}$ is distributed as normal, or $\epsilon_{i,t} \sim N(0, 1)$.

[21] For the goodness-of-fit tests, we make use of the Matlab code *plpva* written by Aaron Clauset. The code is available at http://www.santafe.edu/~aaronc/powerlaws/. We thank Professor Clauset for making this code available. The Matlab script used to estimate the maximum likelihood functions is written by the present authors and available upon request.

According to Taleb (2020), the problem with GARCH-type models is that the parameter estimates would be sample-specific if the fourth moment of $crypto_{i,t}$ was either infinite or did not exist. As a consequence, one cannot rely on these models. Furthermore, Taleb (2020, p. 30) has noted that the (nonparametric) 'empirical distribution' – which in our context is the distribution of realized cryptocurrency volatilities – is not empirical at all because it misrepresents the expected payoffs in the tails. He also emphasized that future maxima are poorly tracked by past data without some intelligent extrapolation. On the other hand, power law functions address this inference problem because the tail exponent of a power law function captures via extrapolation the low-probability deviation not seen in the data, which plays a disproportionately large share in determining the mean.

3.3. Volatility transmission

What are the driving forces of stablecoin volatility? Kyriazis (2019) observed that few academic papers study volatility spillovers among digital currencies. Available evidence suggests that the directional effects of volatility spillovers are mixed. For instance, Katsiampa et al. (2019) found bi-directional effects in the volatility spillovers between Bitcoin–Ethereum, Bitcoin–Litecoin and Ethereum–Litecoin. Similarly, Kumar and Anandarao (2019) explored the dynamics of volatility spillovers concerning the returns of Bitcoin, Ethereum, Ripple, and Litecoin. Their findings indicated that volatility co-movements are considerably more pronounced in bullish market conditions of virtual currencies. On the other hand, Koutmos (2018) examined interdependencies among 18 cryptocurrencies exhibiting high market capitalizations. His findings showed that Bitcoin is the most important cryptocurrency as a generator of volatility spillovers to other high-capitalization cryptocurrencies. Extending these studies, here we explore interdependencies in the volatilities between BTC and stablecoins.

Unlike the aforementioned studies, we do not use GARCH models due to the argument raised in Taleb (2020, p. 50): "GARCH (a method popular in academia) does not work because we are dealing with squares. The variance of the squares is analogous to the fourth moment. We do not know the variance. But we can work very easily with Pareto distributions". Consequently, we propose a novel two-step approach that uses power law distributions (which belong to the class of Pareto distributions accounting for fat-tailed data) to model the underlying probability densities and then utilize the so-called *log–log transformation* (from physics) for making statistical inferences. The log transformation removes or at least reduces by a large margin the skewness of our original data. Subsequently, statistical inferences from these data become valid.

To begin we investigate whether stablecoins and Bitcoin volatility contemporaneously co-move. The following model is employed:

$$btc_t = c + b_1 btc_{t-1} + \sum_{i=1}^{5} h_i stablecoin_{i,t} + \sum_{i=1}^{5} s_i stablecoin_{i,t-1} + u_t, \tag{8}$$

where $btc_t = \ln(\sigma_{BTC,t})$, $stablecoin_{i,t} = \ln\left(\sigma_{Stablecoin_{i,t}}\right)$, and u_t is a white noise process. This model explicitly controls for lagged Bitcoin volatility (btc_{t-1}) as an additional explanatory variable. Using OLS, parameters are estimated as follows (t-statistics in parentheses):

$$btc_t = 0.21^{**}\,(2.32) + 0.19^{***}\,(3.05)\,btc_{t-1} + 0.23^{***}\,(3.15)\,usdt_t + 0.07\,(1.33)\,usdc_t +$$
$$0.20^{***}\,(4.77)\,dai_t + 0.21^{***}\,(2.81)\,busd_t + 0.03\,(0.63)\,tusd_t + 0.02\,(0.21)\,usdt_{t-1}$$
$$-0.16^{***}\,(-2.74)\,usdc_{t-1} + 0.04\,(0.79)\,dai_{t-1} + 0.06\,(0.86)\,busd_{t-1} + 0.02\,(0.36)\,tusd_{t-1}.$$

To assess whether the volatilities of stablecoins and BTC exhibit a contemporaneous effect, we test the hypothesis:

$$H_0: h_1 = h_2 = \cdots = h_5 = 0$$

$$H_1: \textit{at least one } h_i \neq 0, i = \{1, 2, \ldots, 5\}.$$

The Wald test statistic is applied, which under the null hypothesis is asymptotically distributed as chi-square with five degrees of freedom.[22] Since the estimated test statistic $\hat{\lambda}$ has a value of 148.80 and exceeds the 95% critical value corresponding to 11.07 by a large margin (p-value 0.0000), we conclude that Bitcoin volatility and the volatilities of stablecoins contemporaneously co-move.

Next, we test whether the volatilities of stablecoins exhibit any spillover effects on Bitcoin volatility. For this purpose, we test the following hypothesis:

$$H_0: s_1 = s_2 = \cdots = s_5 = 0$$

$$H_1: \textit{at least one} s_i \neq 0, i = \{1, 2, \ldots, 5\}.$$

Again, the Wald test statistic is used (i.e., chi-square distribution with five degrees of freedom). Since the estimated test statistic $\hat{\lambda}$ has a value of 9.48 and does not exceed the 95% critical value corresponding to 11.07 (p-value 0.0961), we conclude that the volatilities of stablecoins do not exhibit any significant spillover effects on Bitcoin volatility.

To test whether Bitcoin volatility exhibits any spillover effects on the volatilities of stablecoins, as shown in Table 4, it is important to note that the volatilities of stablecoins are highly correlated. Consequently, we estimate the following system of equations:

$$usdt_t = a_{1,1} usdt_{t-1} + a_{1,2} btc_t + a_{1,3} btc_{t-1} + e_{1,t} \tag{9.1}$$

[22] It is noteworthy that the estimated residuals vector $\hat{\boldsymbol{u}}$ exhibits a kurtosis of 3.21 and a skewness of 0.14. The Jarque–Bera test cannot reject the null hypothesis of normality (i.e., p-value = 0.5416).

Table 4
Correlation matrix of realized volatilities.

	BTC	USDT	USDC	DAI	BUSD	TUSD
BTC	1					
USDT	0.69***	1				
	(18.14)					
USDC	0.62***	0.80***	1			
	(15.15)	(25.83)				
DAI	0.49***	0.56***	0.57***	1		
	(10.68)	(12.95)	(13.23)			
BUSD	0.74***	0.92***	0.86***	0.55***	1	
	(20.86)	(43.60)	(32.44)	(12.47)		
TUSD	0.68***	0.84***	0.85***	0.56***	0.90***	1
	(17.55)	(29.58)	(30.33)	(12.79)	(38.41)	

We download daily data for Bitcoin (BTC) as well as stablecoins Tether (USDT), USD Coin (USDC), Dai (DAI), Binance USD (BUSD), and TrueUSD (TUSD) from coinmarketcap.com. These data include all available opening, high, low, and closing prices over time. We compute the realized annualized daily volatilities for each cryptocurrency. The realized annualized daily volatilities are computed as,

$$\sigma_{i,t} = \sqrt{T}\sqrt{\left(ln\left(\frac{HIGH_{i,t}}{CLOSE_{i,t}}\right)\cdot ln\left(\frac{HIGH_{i,t}}{OPEN_{i,t}}\right) + ln\left(\frac{LOW_{i,t}}{CLOSE_{i,t}}\right)\cdot ln\left(\frac{LOW_{i,t}}{OPEN_{i,t}}\right)\right)},$$

where $HIGH_{i,t}$, $LOW_{i,t}$, $OPEN_{i,t}$, and $CLOSE_{i,t}$ denote the highest, lowest, opening, and closing price for cryptocurrency i on day t, respectively, $\sigma_{i,t}$ denotes cryptocurrency i's corresponding realized annualized volatility, and $T = 365$ as cryptocurrencies are traded 24/7. This table reports the correlation matrix of the realized annualized daily volatilities for our set of cryptocurrencies. The corresponding t-statistics are given in parentheses.
***Statistically significant at the 1% level.

Table 5
Estimates for system of multiple equations.

	usdt	usdc	dai	busd	tusd
constant	−0.23***	−0.52***	−0.63***	−0.34***	−0.56***
	(−5.93)	(−7.61)	(−6.54)	(−3.84)	(−10.52)
lagged stablecoin	0.79***	0.66***	0.28***	0.78***	0.58***
	40.73	24.29	4.99	26.81	23.96
btc	0.50***	0.42***	0.43***	0.50***	0.42***
	14.60	8.61	5.75	7.55	10.92
btc_{t-1}	−0.36***	−0.32***	−0.09	−0.44***	−0.27***
	(−9.78)	(−6.65)	(−1.14)	(−6.65)	(−6.88)
R-squared	0.64	0.51	0.22	0.68	0.45

Tests of whether Bitcoin volatility exhibits any spillover effects on the volatilities of stablecoins. The following system of equations is estimated:

$$usdt_t = a_{1,1}usdt_{t-1} + a_{1,2}btc_t + a_{1,3}btc_{t-1} + e_{1,t}$$
$$usdc_t = a_{2,1}usdc_{t-1} + a_{2,2}btc_t + a_{2,3}btc_{t-1} + e_{2,t}$$
$$dai_t = a_{3,1}dai_{t-1} + a_{3,2}btc_t + a_{3,3}btc_{t-1} + e_{3,t}$$
$$busd_t = a_{4,1}busd_{t-1} + a_{4,2}btc_t + a_{4,3}btc_{t-1} + e_{4,t}$$
$$tusd_t = a_{5,1}tusd_{t-1} + a_{5,2}btc_t + a_{5,3}btc_{t-1} + e_{5,t},$$

where btc_t, $usdt_t$, dai_t, $busd_t$, and $tusd_t$ denote the natural logarithms of Bitcoin (BTC), Tether (USDT), USD Coin (USDC), Dai (DAI), Binance USD (BUSD), and TrueUSD (TUSD), $e_{1,t}$, $e_{2,t}$, …, $e_{5,t}$ denote contemporaneously correlated error processes that have a 5 × 5 covariance matrix denoted as $\mathbf{\Sigma}$. The system of equations was estimated using the Seemingly Unrelated Regression (SUR) estimation technique. This table shows the results for these equations with t-statistics in parentheses.
***Statistically significant at the 1% level.

$$usdc_t = a_{2,1}usdc_{t-1} + a_{2,2}btc_t + a_{2,3}btc_{t-1} + e_{2,t} \tag{9.2}$$

$$dai_t = a_{3,1}dai_{t-1} + a_{3,2}btc_t + a_{3,3}btc_{t-1} + e_{3,t} \tag{9.3}$$

$$busd_t = a_{4,1}busd_{t-1} + a_{4,2}btc_t + a_{4,3}btc_{t-1} + e_{4,t} \tag{9.4}$$

$$tusd_t = a_{5,1}tusd_{t-1} + a_{5,2}btc_t + a_{5,3}btc_{t-1} + e_{5,t}, \tag{9.5}$$

where $e_{1,t}$, $e_{2,t}$, …, $e_{5,t}$ denote contemporaneously correlated error processes that have a 5 × 5 covariance matrix $\mathbf{\Sigma}$. We see from Eqs. (9.1) to (9.5) that each equation controls for own lagged volatility. After estimating this system using the Seemingly Unrelated Regression (SUR) estimation technique, Table 5 shows that, if Bitcoin volatility increases by 1%, the volatilities of stablecoins increase contemporaneously between 0.41% and 0.50%. In view of the positive correlation of the volatility processes, these findings are not surprising. Also, the volatilities of stablecoins exhibit positive autocorrelations due to statistically significant lags at all critical levels. Interestingly, the lagged volatility of Bitcoin appears to have an impact on the volatility of stablecoins also. Another interesting feature is that the coefficient of determination (R^2) values from regression equations (9.1)–(9.5) are 0.64 (USDT), 0.51 (USDC), 0.22 (DAI), 0.68 (BUSD), and 0.45 (TUSD), indicating that our multiple equation model based on using *log–log transformations* of the data is able to explain, on average, half of the variation in the realized stablecoin volatilities.

Finally, to assess whether Bitcoin volatility is Granger-causal for the volatilities of stablecoins, we test the following hypothesis:

$$H_0 : a_{1,3} = a_{2,3} = \cdots = a_{5,3} = 0$$

$$H_1 : \textit{at least one } a_{i,3} \neq 0, i = \{1, 2, \ldots, 5\}.$$

As before, the Wald test statistic is utilized (i.e., chi-square distributed with five degrees of freedom). Since the estimated test statistic $\hat{\lambda}$ has a value of 176.14 and exceeds the 95% critical value corresponding to 11.07 by a large margin (p-value 0.0000), we conclude that Bitcoin volatility has a Granger-causal effect on the volatilities of stablecoins. Also, if lagged Bitcoin volatility increases, a negative effect occurs on the volatilities of stablecoins with decreases on the day that follows, and vice versa.

4. Discussion

4.1. Why choose power laws for modeling volatility processes?

Unlike prior studies that use GARCH models, we modeled the probability density of realized volatility processes using power laws. One reason for this approach is that valid statistical inference based on GARCH models requires that: (*i*) the theoretical variance exists, and (*ii*) the variance can be reliably estimated. Due to fat-tailed data, these conditions are typically not satisfied.[23] For instance, even if the theoretical mean exists, the sample mean estimator may suffer from a persistent small sample effect. That is, the law of large numbers (LLN) works too slowly given data limitations, such that it is not possible to reliably estimate the mean.

Previous authors have supported the use of power laws in empirical analyses. For example, Taleb (2020, p. 91) observed: "There are a lot of theories on why things should be power laws, as sort of exceptions to the way things work probabilistically. But it seems that the opposite idea is never presented: power laws should be the norm, and the Gaussian a special case". Also, Lux and Alfarano (2016, p. 4) commented on the application of power laws to financial market data: "… power laws in returns and in volatility seem to be intimately related: none of them was ever observed without the other and it, therefore, seems warranted to interpret them as the joint essential characteristics of financial data". Previous work by Lux and Ausloos (2002) employed power laws to model the autocovariance function of absolute returns, which can be considered as a proxy for volatility. Also, Calvet and Fisher (2004) and Lux et al. (2014) showed that these models often outperform GARCH models in terms of forecasting volatility.

4.2. Why not employ the Kolmogorov–Smirnov approach?

According to Eq. (6), estimating the power law exponent depends on the choice of $\hat{x}_{MIN}$. Clauset et al. (2009) discussed different popular approaches for identifying $\hat{x}_{MIN}$ and chose the minimization of the Kolmogorov–Smirnov distance for this purpose. As noted earlier, this approach may be accurate when data is well-behaved. To illustrate this issue, we plot Bitcoin volatility in Fig. A.1 in the Appendix based on the empirical distribution of the Kolmogorov–Smirnov (KS) distance on the y-axis and the corresponding $\hat{x}_{MIN}$ on the x-axis. We see from this figure that the KS distance is minimized for $\hat{x}_{MIN} = 1.0950$ with corresponding $D = 0.0371$ and $\hat{\alpha} = 3.4643$. In Fig. A.2 in the Appendix, we plot on the y-axis the empirical density function for Bitcoin volatility and on the x-axis the theoretical density function for the power law model with $\left(\hat{\alpha}, \hat{x}_{MIN}\right) = (3.4643, 1.0950)$. If the data fit was perfect, all observations would lie on the 45 degree line. The correlation between the empirical and theoretical probability density function is 0.9994. Predictably, the goodness-of-fit test suggests a p-value of 0.5027, which implies that we cannot reject the power law null hypothesis. This result provides strong evidence for an almost perfect empirical fit. The chosen power law model describes the data generating process almost perfectly. This finding supports earlier findings by Liu et al. (1999), who documented (as mentioned earlier) that the asymptotic behavior of the probability density function for the S&P 500 index is best described by a power law distribution. Our results indicate that this is the case for the largest virtual currency market also.

If the data is not well-behaved, matters become more complicated. As an example, we consider the case of stablecoin DAI. In Fig. A.3 in the Appendix, we plot the empirical distribution of the Kolmogorov–Smirnov distance on the y-axis and the corresponding $\hat{x}_{MIN}$ on the x-axis. The Kolmogorov–Smirnov distance is minimized for $\hat{x}_{MIN} = 0.4896$ with corresponding $D = 0.0708$ and $\hat{\alpha} = 4.0077$. In Fig. A.4 , we plot on the y-axis the empirical density function and on the x-axis the theoretical density function for the power law model with $\left(\hat{\alpha}, \hat{x}_{MIN}\right) = (4.0077, 0.0708)$. Even though the correlation between empirical and theoretical density function is 0.9924, given the parameter vector $\left(\hat{\alpha}, \hat{x}_{MIN}\right) = (4.0077, 0.0708)$ associated with the minimum D, we observe from Table 3 that the power law hypothesis is rejected for $\alpha > 3.00$. This result implies that the empirical data are not in line with a power law process defined by this parameter vector. Since the data strongly indicate the presence of fat tails, we search for a parameter vector $\left(\hat{\alpha}, \hat{x}_{MIN}\right)$ that defines a power law process that mimics the empirical data.

Hence, our approach does not rely on initially minimizing the KS distance and then employing the goodness-of-fit test. Instead, we use the goodness-of-fit test directly in a trial-and-error procedure that has the objective of searching for the combination $\left(\hat{\alpha}, \hat{x}_{MIN}\right)$

[23] Our previous analyses indicated that the theoretical variances of the volatility processes of stablecoins do not exist.

for which we cannot reject the power law null hypothesis for the first time. Specifically, since we know that we cannot reject the power law null hypothesis for $\hat{\alpha} = 2.50$, we search for the optimal candidate for $\hat{x}_{MIN}$ that produces an $\hat{\alpha}$ in the interval $[2.50, 3.00]$ that does not reject the power law null hypothesis for the first time. This procedure yields the combination $\left(\hat{\alpha}, \hat{x}_{MIN}\right) = (2.8699, 0.2268)$ with corresponding $D = 0.1351$. In Fig. A.5 in the Appendix, we plot the empirical density function for DAI volatility on the y-axis and on the x-axis the theoretical density function for the power law model with $\left(\hat{\alpha}, \hat{x}_{MIN}\right) = (2.8699, 0.2268)$. The correlation between the empirical and theoretical density functions is 0.9905, which indicates an excellent data fit. Since the goodness-of-fit test suggests a p-value of $0.0534 > 0.0500$, we do not reject the power law null hypothesis for our chosen parameter combination. It is noteworthy that, the lower the alpha is chosen (i.e., the fatter the tail of the underlying power law distribution), the greater the severity of Black Swan events.

The latter inference is consistent with Taleb (2012), who emphasized that the underestimation of tail events is a serious issue in risk management. Although our approach exhibits a higher Kolmogorov–Smirnov distance than the global minimum ($D = 0.0708$), it yields a parameter combination that (*i*) does not reject the power law null hypothesis and (*ii*) allows for fatter tails due to allowing a lower optimum exponent than the exponent estimated via the minimum of the global KS distance. Moreover, it is important to recognize that the minimum of the global KS distance does not necessarily yield power law exponents for which the goodness-of-fit test does not reject the power law null hypothesis.

As an example, consider the power law exponents associated with the global minima for the corresponding KS distances with respect to volatilities of USDT, USDC, BUSD, and TUSD equal to 3.6481, 3.5102, 3.2430 and 3.3089, respectively. Taken at face value, these findings suggest that the volatilities of these stablecoins exhibit stochastic properties similar to Bitcoin volatility. However, the goodness-of-fit test clearly rejects the power law null hypothesis for these data series, whereas Table 3 shows that our approach yields parameter estimates that are in line with the theoretical power law distribution. That is, using our approach to choose the optimal power law exponent, the power law null hypothesis cannot be rejected for all data series. We infer that the $\hat{x}_{MIN}$ associated with the global minimum is not necessarily the optimal $\hat{x}_{MIN}$ in the sense that it generates the theoretical probability distribution of the underlying data. This inference is especially relevant to erratic data as seen by visual inspection of the corresponding $\hat{\alpha}/\hat{x}_{MIN}$-function.

4.3. How do stablecoins' zero-volatilities affect our results?

We note that the realized volatility is zero for 806 (out of 2,093), 90 (out of 777), 58 (out of 367), 35 (out of 430), and 106 (out of 993) cases for USDT, USDC, DAI, BUSD, and TUSD, respectively. This comes as no surprise as stablecoin prices are expected to be exactly 1 USD (or at least constant which implies that the realized volatility is thus zero). Since the realized volatility distributions appear to be concentrated at zero for some stablecoins, one could question whether zero-volatilities induce the heavy-tailedness reflected in the kurtosis. When applying power law functions to these data, are zero-volatilities the main driver of the infinite variances of the volatility processes?

First, we explored this issue by excluding all zero-volatilities and again calculating the kurtosis values for all stablecoins. The kurtosis values for USDT, USDC, DAI, BUSD, and TUSD excluding zero-volatilities were estimated at 37.45, 19.01, 15.38, 41.16, and 179.74, respectively. Comparing these figures with the kurtoses using the whole datasets, as reported in Table 2, we found the differences to be negligible. Second, because the support of the power law function is only defined for $x \geq x_{MIN}$, an important issue in applying power law functions for modeling probability densities is the minimum value x_{MIN}. As observed by Clauset et al. (2009), for values $x < x_{MIN}$, a different distribution is employed. To illustrate this issue, let us consider USDT. Given our estimates, the overall probability density function for USDT is defined as,

$x = 0$ with probability $p = p_1$,

$0 < x < x_{MIN}$ with probability $p = p_2$,

$(\alpha - 1)\, x_{MIN}^{\alpha-1} x^{-\alpha}$ with probability $p = 1 - p_1 - p_2$,

where $\left(p_1 + p_2\right) < 1$. Knowing that the realized volatility is zero for 806 (out of 2,093) cases for USDT, we find that $p_1 = 0.39$. Moreover, for 316 (out of 2,093) cases the realized volatility for USDT fulfills $0 < x < x_{MIN}$ with $x_{MIN} = 0.1752$. Hence, we find that $p_2 = 0.15$ and, subsequently, the probability of x being in the tail corresponds to a probability of $p = 0.46$. In this regard, there are two important aspects. First, zero-volatilities are not part of the power law process, as $x_{MIN} > 0$ for all realized stablecoin volatilities (see Panel C of Table 3). Second, from Eq. (6), the maximum likelihood estimation procedure does not incorporate values for which $x < x_{MIN}$. Therefore, we conclude that from a statistical point of view zero-volatilities are not the main driver of the infinite variances of realized stablecoin volatilities.

4.4. How do our results compare to earlier studies?

Using the framework of Diebold and Yilmaz (2009) to measure interdependencies among 18 different cryptocurrencies, Koutmos (2018) found that Bitcoin is the dominant transmission catalyst for shocks in the other sampled cryptocurrencies. Using a different research methodology and different set of cryptocurrencies (stablecoins), our study shows that Bitcoin volatility is not only contemporaneously responding to stablecoin volatilities but that Bitcoin volatility is causal in a Granger-sense. Moreover, a recent study by Baur and Hoang (2021b) found evidence that stablecoins are a safe haven against large negative price changes in Bitcoin.

Our study finds that an increase in lagged Bitcoin volatility decreases the future volatilities of stablecoins, which confirms Baur and Hoang's safe haven argument. The authors also showed that returns are significantly correlated with Bitcoin returns, which implies excess volatility of stablecoins rendering them unstable.

Our findings confirm those of Baur and Hoang (2021b) that stablecoins are not stable. Unlike Baur and Hoang (2021b), however, we draw this inference using a different research methodology. Since realized volatilities are extremely fat-tailed processes, we followed a different stream of literature using power law functions to model the stochastic processes (viz., Lux and Alfarano, 2016; Lux and Ausloos, 2002; Calvet and Fisher, 2004; Lux et al., 2014; Liu et al., 1999). In this regard, Calvet and Fisher (2004) and Lux et al. (2014) showed that power law models typically outperform GARCH models in terms of forecasting financial market volatility. We show that, because the power law exponents are significantly < 3, the variances of our realized stablecoin volatilities are mathematically not defined. One manifestation of this result is that the volatility processes of stablecoins are more prone to extreme events dominating the overall distribution. Since the variances of our realized stablecoin volatilities do not exist, sample estimates of volatilities are uninformative because we do not observe the true values in finite samples. Note also that the stablecoins considered here are pegged to the U.S. dollar and should by definition (at least theoretically) exhibit a volatility of ≈ 0. By contrast, Bitcoin is not supposed to exhibit a volatility of ≈ 0, as it is not pegged to any underlying. The price dynamics are explicitly driven by the demand side with the supply side fixed. Even though Bitcoin is highly volatile, because the power law exponent is statistically significantly > 3, the variance of realized Bitcoin volatility is mathematically defined, and hence, the realized volatility of Bitcoin is computable (and informative in a finite sample).

Finally, our findings contradict Katsiampa et al. (2019), who found bi-directional effects in volatility spillovers between some large cap cryptocurrencies. This difference can be attributed to the fact that stablecoins belong to a separate class of cryptocurrencies designed to be pegged to an underlying national currency such as the U.S. dollar.[24]

4.5. Are our results robust?

Given our results, one could hypothesize that, even if Bitcoin affects the volatility of stablecoins, there should be a common factor that drives the volatilities. To investigate this issue, we employ Diebold and Yilmaz's (2009) volatility spillover index. Following Grobys (2015) and Grobys and Vähämaa (2020), we apply this methodology directly to the realized volatilities. Defining a 6×1 vector $\boldsymbol{Y}_t$ as $\boldsymbol{Y}_t = \left(BTC_t, USDT_t, USDC_t, DAI_t, BUSD_t, TUSD_t\right)'$, we employ the following vector-autoregression (VAR) model:

$$\boldsymbol{Y}_t = \boldsymbol{c} + \boldsymbol{A}_1 \boldsymbol{Y}_{t-1} + \cdots + \boldsymbol{A}_p \boldsymbol{Y}_{t-p} + \boldsymbol{u}_t, \tag{10}$$

where $\boldsymbol{A}_1, \ldots, \boldsymbol{A}_p$ are 6×6 parameter matrices, and the error term $\boldsymbol{u}_t$ is assumed to be distributed as $\boldsymbol{u}_t \sim \left(0, \boldsymbol{\Sigma}_{\boldsymbol{u}}\right)$ in which $\boldsymbol{\Sigma}_{\boldsymbol{u}}$ denotes the corresponding covariance matrix. Moreover, $\boldsymbol{c}$ is a 6×1 vector containing the constant terms. We implement an iteratively updated rolling time-window of 60 days and three different lag orders of $p = (1, 2, 4)$. The estimated parameter matrices $\hat{\boldsymbol{A}}_1, \ldots, \hat{\boldsymbol{A}}_{t-p}$ from Eq. (10) are used to model the corresponding Wold moving average (MA) representation:

$$\boldsymbol{Y}_t = \boldsymbol{\Phi}_0 \boldsymbol{u}_t + \boldsymbol{\Phi}_1 \boldsymbol{u}_{t-1} + \boldsymbol{\Phi}_2 \boldsymbol{u}_{t-2} + \cdots, \tag{11}$$

where $\boldsymbol{\Phi}_0 = \boldsymbol{I}_{6,6}$ and

$$\boldsymbol{\Phi}_s = \sum_{j=1}^{s} \boldsymbol{\Phi}_{s-j} \widehat{\boldsymbol{A}_j}, \quad s = 1, 2, \ldots \tag{12}$$

is compounded recursively. We then use a Cholesky decomposition of the covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{u}}$, which we define as matrix $\boldsymbol{D}$. If $\boldsymbol{D}$ is a lower triangular matrix, such that $\boldsymbol{\Sigma}_{\boldsymbol{u}} = \boldsymbol{D}\boldsymbol{D}'$, then the orthogonalized shocks are given by $\boldsymbol{\varepsilon}_t = \boldsymbol{D}^{-1}\boldsymbol{u}_t$. Consequently, we obtain:

$$\boldsymbol{Y}_t = \boldsymbol{\Psi}_0 \boldsymbol{\varepsilon}_t + \boldsymbol{\Psi}_1 \boldsymbol{\varepsilon}_{t-1} + \boldsymbol{\Psi}_2 \boldsymbol{\varepsilon}_{t-2} + \cdots, \tag{13}$$

where $\boldsymbol{\Psi}_i = \boldsymbol{\Phi}_i \boldsymbol{D} (i = 0, 1, 2, \ldots)$. Since $\boldsymbol{\Psi}_0 = \boldsymbol{D}$ is a lower triangular, a shock occurring on the first variable has in instantaneous effect on the second variable in the system. Due to our chosen ordering, matrices $\boldsymbol{\Psi}_0$ and $\boldsymbol{\Psi}_1$ are defined simply as:

$$\boldsymbol{\Psi}_0 = \begin{bmatrix} \Psi_{0,11} & \Psi_{0,12} & \Psi_{0,13} & \Psi_{0,14} & \Psi_{0,15} & \Psi_{0,16} \\ \Psi_{0,21} & \Psi_{0,22} & \Psi_{0,23} & \Psi_{0,24} & \Psi_{0,25} & \Psi_{0,26} \\ \Psi_{0,31} & \Psi_{0,32} & \Psi_{0,33} & \Psi_{0,34} & \Psi_{0,35} & \Psi_{0,36} \\ \Psi_{0,41} & \Psi_{0,42} & \Psi_{0,43} & \Psi_{0,44} & \Psi_{0,45} & \Psi_{0,46} \\ \Psi_{0,51} & \Psi_{0,52} & \Psi_{0,53} & \Psi_{0,54} & \Psi_{0,55} & \Psi_{0,56} \\ \Psi_{0,61} & \Psi_{0,62} & \Psi_{0,63} & \Psi_{0,64} & \Psi_{0,65} & \Psi_{0,66} \end{bmatrix}$$

[24] Following earlier research investigating volatility spillover effects as a potential risk factor in explaining uncertainties in traditional foreign exchange markets (Cáceres, 2003; Melvin and Melvin, 2003; Baruník et al., 2017), Baur and Hoang (2021a) found that the correlation of trading volumes between stablecoins and Bitcoin is very high. The authors argued that the positive correlation between trading volumes of stablecoins and Bitcoin volatility indicates that stablecoins not only facilitate Bitcoin trading but contribute to Bitcoin volatility also.

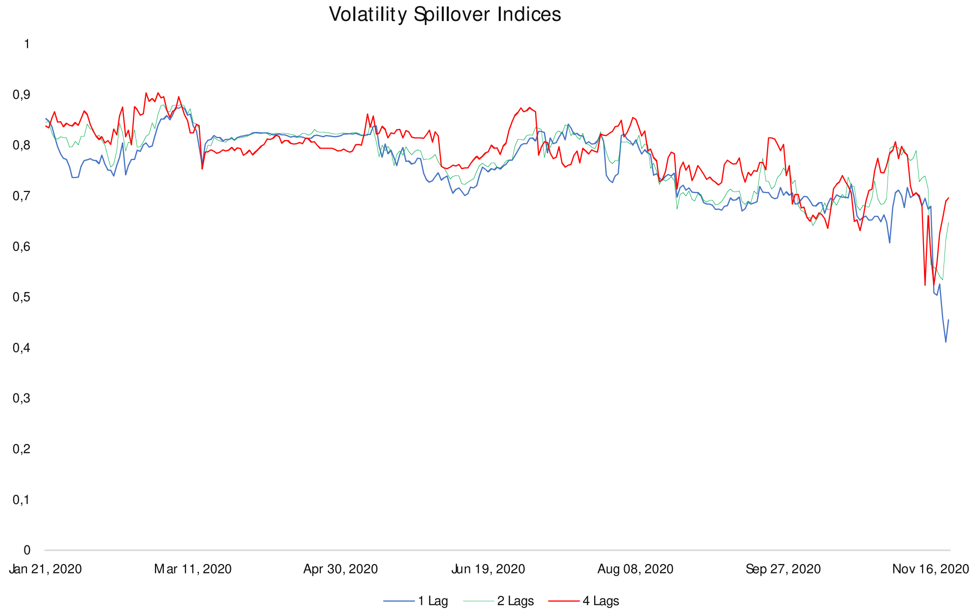

Fig. 3. Volatility spillover indices.
This figure plots the volatility spillover indices for the realized annualized daily volatilities for Bitcoin, Tether, USD Coin, Dai, Binance USD, and TrueUSD over time. Using
$\boldsymbol{Y}_t = \boldsymbol{\Phi}_0\boldsymbol{u}_t + \boldsymbol{\Phi}_1\boldsymbol{u}_{t-1} + \boldsymbol{\Phi}_2\boldsymbol{u}_{t-2} + \cdots$, where
$\boldsymbol{Y}_t = \left(BTC_t, USDT_t, USDC_t, DAI_t, BUSD_t, TUSD_t\right)'$, $\boldsymbol{\Phi}_0 = \boldsymbol{I}_{6,6}$ and $\boldsymbol{\Phi}_s = \sum_{j=1}^{s} \boldsymbol{\Phi}_{s-j}\widehat{\boldsymbol{A}_j}$, provided
$\boldsymbol{Y}_t = \boldsymbol{c} + \boldsymbol{A}_1\boldsymbol{Y}_{t-1} + \cdots + \boldsymbol{A}_p\boldsymbol{Y}_{t-p} + \boldsymbol{u}_t$, where
$\boldsymbol{A}_1, \ldots, \boldsymbol{A}_p$ are 6×6 parameter matrices, $\boldsymbol{c}$ defines a 6×1 vector of constants, the error term $\boldsymbol{u}_t$ is assumed to be distributed as $\boldsymbol{u}_t \sim \left(\boldsymbol{0}, \boldsymbol{\Sigma}_u\right)$, where $\boldsymbol{\Sigma}_u$ denotes the corresponding covariance matrix, the spillover index for lag order $p = (1, 2, 4)$ can then be computed employing the matrices $\boldsymbol{\Psi}_0$ in association with $\boldsymbol{\Psi}_1$ estimated by
$\boldsymbol{Y}_t = \boldsymbol{\Psi}_0\boldsymbol{\varepsilon}_t + \boldsymbol{\Psi}_1\boldsymbol{\varepsilon}_{t-1} + \boldsymbol{\Psi}_2\boldsymbol{\varepsilon}_{t-2} + \cdots$, where
$\boldsymbol{D}$ is a lower triangular matrix computed as the Cholesky decomposition of the covariance matrix $\boldsymbol{\Sigma}_u$ such that $\boldsymbol{\Sigma}_u = \boldsymbol{D}\boldsymbol{D}'$, $\boldsymbol{\Psi}_i = \boldsymbol{\Phi}_i\boldsymbol{D}(i = 0, 1, 2, \ldots)$, and $\boldsymbol{\Psi}_0 = \boldsymbol{D}$. Orthogonalized shocks are given by $\boldsymbol{\varepsilon}_t = \boldsymbol{D}^{-1}\boldsymbol{u}_t$. The realized volatility spillover index for the one-step-ahead forecast is then constructed by summing up all elements above and below the main diagonals of $|\boldsymbol{\Psi}_0|$ and $|\boldsymbol{\Psi}_1|$ and dividing by the total sum of all elements in the matrices $|\boldsymbol{\Psi}_0|$ and $|\boldsymbol{\Psi}_1|$. The sample period is from November 22, 2019 to November 22, 2020.

and

$$\boldsymbol{\Psi}_1 = \begin{bmatrix} \Psi_{1,11} & \Psi_{1,12} & \Psi_{1,13} & \Psi_{1,14} & \Psi_{1,15} & \Psi_{1,16} \\ \Psi_{1,21} & \Psi_{1,22} & \Psi_{1,23} & \Psi_{1,24} & \Psi_{1,25} & \Psi_{1,26} \\ \Psi_{1,31} & \Psi_{1,32} & \Psi_{1,33} & \Psi_{1,34} & \Psi_{1,35} & \Psi_{1,36} \\ \Psi_{1,41} & \Psi_{1,42} & \Psi_{1,43} & \Psi_{1,44} & \Psi_{1,45} & \Psi_{1,46} \\ \Psi_{1,51} & \Psi_{1,52} & \Psi_{1,53} & \Psi_{1,54} & \Psi_{1,55} & \Psi_{1,56} \\ \Psi_{1,61} & \Psi_{1,62} & \Psi_{1,63} & \Psi_{1,64} & \Psi_{1,65} & \Psi_{1,66} \end{bmatrix}$$
.

Since volatility spillover indices measure economic magnitudes, we compound for each matrix $\boldsymbol{\Psi}_0$ and $\boldsymbol{\Psi}_1$ the corresponding matrix $|\boldsymbol{\Psi}_0|$ and $|\boldsymbol{\Psi}_1|$.[25] The realized volatility spillover index for the one-step-ahead forecast is then constructed by summing up all elements above and below the main diagonals of $|\boldsymbol{\Psi}_0|$ and $|\boldsymbol{\Psi}_1|$ and dividing by the total sum of all elements in the matrices $|\boldsymbol{\Psi}_0|$ and $|\boldsymbol{\Psi}_1|$.[26] This relation approaches unity if and only if the volatility processes of both series are driven by spillovers only, whereas it approaches zero if and only if the volatility processes are driven by their own past volatilities. As in Grobys and Vähämaa (2020), we use a forecast horizon of $h = 1$, which corresponds to one day given our data. Models are updated daily in the sample period from November 22, 2019 to November 22, 2020 (due to data availability of DAI). Fig. 3 illustrates the time series evolutions of the second moments' spillover indices over the sample period.

Using $p = 4$ lags in association with a forecast horizon of $h = 1$ for implementing the iteratively updated VAR models, we observe from Fig. 3 that the spillover index varies between a minimum value of 0.52 and a maximum value of 0.90. The average value of

[25] Our approach departs from the methodology in Diebold and Yilmaz (2009). Because they utilized returns, the variance-error-decomposition was compounded, wherein elements of matrices are squared such that all elements are positive for measuring the economic magnitudes of spillovers. By contrast, following Grobys (2015), we directly employ the realized volatilities as input variables.

[26] For example, the contribution of the volatility from Bitcoin volatility to the one-step-ahead forecast of volatility of Tether is given by $|\Psi_{0,21}| + |\Psi_{1,21}|$. Note that volatility spillover indices do not provide insights concerning the direction of correlations. The spillover index approaches 1 in both cases (i.e., high levels of negative or positive correlations across the realized cryptocurrency volatilities).

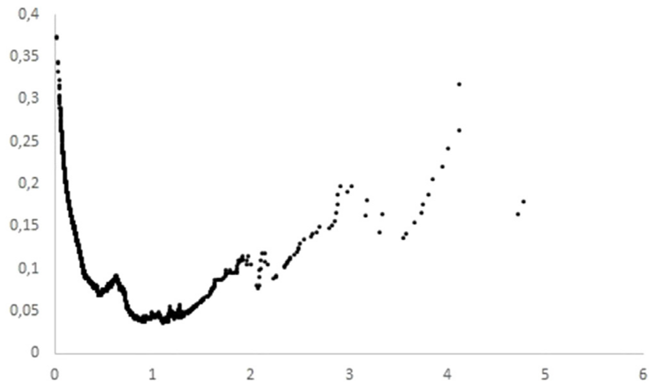

Fig. A.1. Kolmogorov–Smirnov distances depending on x_{MIN} for Bitcoin volatility.
We used maximum likelihood estimation (MLE) for the realized annualized daily volatility of Bitcoin and estimated $\hat{\alpha}$ as a function of x_{MIN} as, $\hat{\alpha} = 1 + N\left(\sum_{i=1}^{N} \ln\left(\frac{x_i}{x_{MIN}}\right)\right)^{-1}$, where $\hat{\alpha}$ denotes the MLE estimator, x_i is the realized annualized daily volatility of the respective cryptocurrency, provided $x_i \geq x_{MIN}$, and N denotes the number of observations for which $x_i \geq x_{MIN}$ is satisfied. Next, we employ the Kolmogorov–Smirnov approach to choose the optimal value for $\hat{x}_{MIN}$. This statistic is simply the maximum distance D between the data and fitted CDFs defined as, $D = MAX_{x \geq x_{MIN}} |S(x) - P(x)|$,
where $S(x)$ is the CDF of the data for the observation with value at least x_{MIN}, and $P(x)$ is the CDF for the power law model that best fits the data in the region $x \geq x_{MIN}$. The estimate of the x_{MIN} is the value of x_{MIN} that minimizes D. In this figure we plot the empirical distribution of the Kolmogorov–Smirnov distance (y-axis) depending on the x_{MIN} (x-axis).

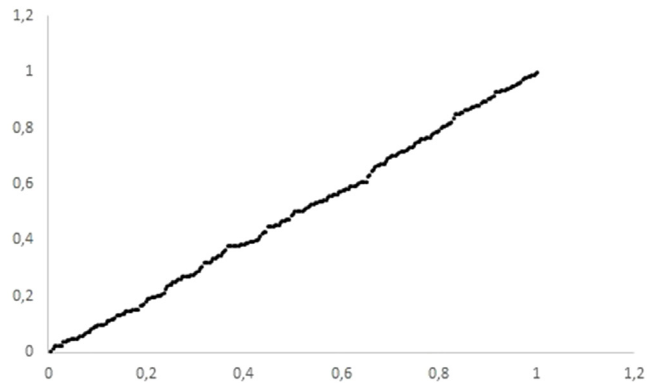

Fig. A.2. Empirical and theoretical density function at the optimum for Bitcoin volatility.
Plot of the empirical density function for Bitcoin volatility on the y-axis and the theoretical density function for the power law model with $(\alpha, x_{MIN}) = (3.4643, 1.0950)$ on the x-axis. If the data fit was perfect, all observations would lie on the 45 degree line.

the spillover index is 0.79. Since the spillover index is strictly above 0.50, we interpret this as evidence supporting our previous finding that the volatilities of our set of cryptocurrencies share a common factor. Notably, using one or two lags in the VAR model as additional robustness checks strongly supports our finding, that is, irrespective of which lag-order we use for the underlying VAR model specifications, there is strong evidence for a common factor that drives the volatilities. In view of earlier findings, we infer that the Bitcoin volatility is the common factor.

5. Conclusion

This paper sought to investigate the volatility processes of stablecoins and their potential stochastic interdependencies with Bitcoin volatility. We employed an established measure of daily volatility using high, low, open, and closing prices for a number of cryptocurrencies. Our findings indicated that Bitcoin volatility is stable in the statistical sense that a theoretical variance exists. While Bitcoin volatility is relatively well-behaved, the volatilities of stablecoins are more erratic. For this reason, we cannot utilize Clauset et al.'s (2009) approach for estimating the power law exponent $\hat{\alpha}$ by using the value of $\hat{x}_{MIN}$ that minimizes the (global) Kolmogorov–Smirnov distance. Instead, we proposed an alternative (local) approach that is based on trial-and-error to search for the parameter combination ($\hat{\alpha}$, $\hat{x}_{MIN}$) for which the power law null hypothesis cannot be rejected.

Our empirical results indicated the volatilities of stablecoins are statistically unstable due to infinite theoretical variances. Why are stablecoins unstable? Whereas Bitcoin is not pegged to any underlying, the stablecoins under study here are pegged to the U.S. dollar; hence the volatility of these cryptocurrencies is constrained. According to Taleb (2012, p. 106), if volatility is artificially suppressed, the system can become not only fragile but visibly reveal little or no risks even though such risks are latently growing.

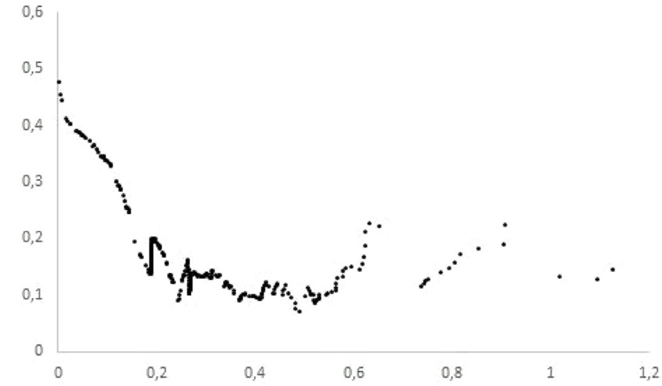

Fig. A.3. Kolmogorov–Smirnov distances depending on x_{MIN} for DAI volatility.
We used maximum likelihood estimation (MLE) for the realized annualized daily volatility of Dai and estimated $\hat{\alpha}$ as a function of x_{MIN} as, $\hat{\alpha} = 1 + N\left(\sum_{i=1}^{N} \ln\left(\frac{x_i}{x_{MIN}}\right)\right)^{-1}$, where $\hat{\alpha}$ denotes the MLE estimator, x_i is the realized annualized daily volatility of the respective cryptocurrency, provided $x_i \geq x_{MIN}$, and N denotes the number of observations for which $x_i \geq x_{MIN}$ is satisfied. Next, we employ the Kolmogorov–Smirnov approach to choose the optimal value for $\hat{x}_{MIN}$. This statistic is simply the maximum distance D between the data and fitted CDFs defined as, $D = MAX_{x \geq x_{MIN}} |S(x) - P(x)|$, where $S(x)$ is the CDF of the data for the observation with value at least x_{MIN}, and $P(x)$ is the CDF for the power law model that best fits the data in the region $x \geq x_{MIN}$. The estimate of the x_{MIN} is the value of x_{MIN} that minimizes D. In this figure we plot the empirical distribution of the Kolmogorov–Smirnov distance (y-axis) depending on the x_{MIN} (x-axis).

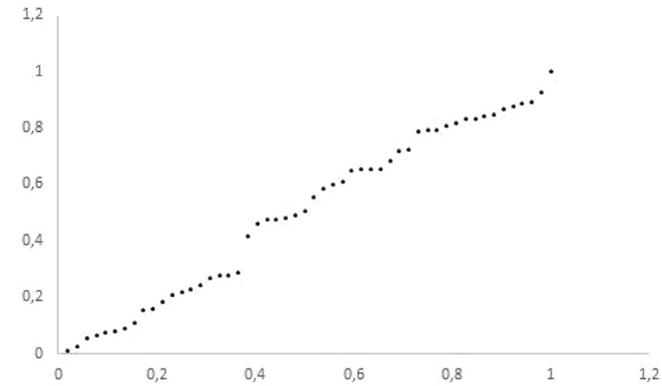

Fig. A.4. Empirical and theoretical density function at the KS optimum for DAI volatility.
Plot of the empirical density function for DAI volatility on the y-axis and the theoretical density function for the power law model with $(\alpha, x_{MIN}) = (4.0077, 0.0708)$ on the x-axis. If the data fit was perfect, all observations would lie on the 45 degree line.

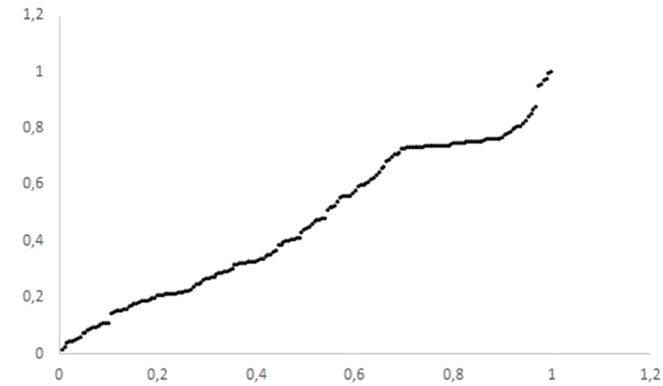

Fig. A.5. Empirical and theoretical density function at the trial-and-error optimum for DAI volatility.
Plot of the empirical density function for DAI volatility on the y-axis and the theoretical density function for the power law model with $(\alpha, x_{MIN}) = (2.8699, 0.2268)$ on the x-axis. If the data fit was perfect, all observations would lie on the 45 degree line.

Next, we found that Bitcoin volatility exhibits volatility spillover effects on stablecoins in a Granger-sense. Previous work by Lyons and Viswanath-Natraj (2019) found that on average an increase in the volatility of Bitcoin trading had a positive effect on the Tether price that was particularly pronounced in turbulent periods. Extending their analyses, our study examined interdependencies in the second moment and found a negative relation in the volatility processes — that is, as lagged Bitcoin volatility decreases, the volatilities of stablecoins (including Tether) tends to increase. This effect was statistically significant across different stablecoins. Based on our empirical results, we conclude that Bitcoin volatility is a fundamental factor that drives the volatilities of stablecoins. Since our research showed that our models explain, on average, about half of the variation of the realized stablecoin volatilities, what remaining forces drive stablecoin volatility? Further research is recommended on the identification of sources of cryptocurrency uncertainty that drive this process. Also, future research is encouraged to elaborate more on the negative relationship between lagged Bitcoin volatility and stablecoin volatility.

CRediT authorship contribution statement

Klaus Grobys: Conceptualization, Data, Methodology, Software, Writing - original draft, Validation. **Juha Junttila:** Supervision, Writing - review & editing, Methodology, Validation. **James W. Kolari:** Supervision, Writing - review & editing. **Niranjan Sapkota:** Visualization, Investigation, Writing - review & editing.

Appendix

See Figs. A.1–A.5.

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